Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: applications and interpretation
Higher level
Paper 3

20 May 2026

Zone A afternoon | **Zone B** afternoon | **Zone C** afternoon

1 hour 15 minutes

Instructions to students

- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all the questions in the answer booklet provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: applications and interpretation HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[55 marks]**.

2226 – 5222

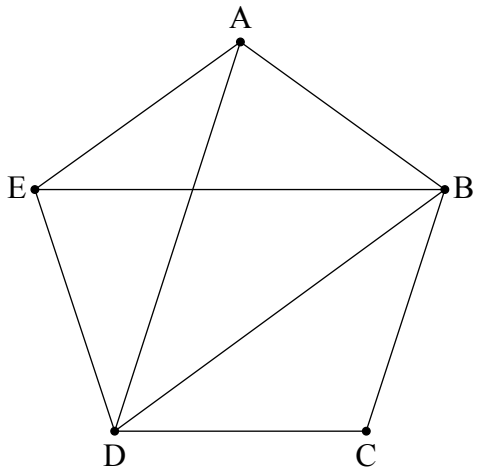
9 pages

Answer **both** questions in the answer booklet provided. Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 29]

This question is about different routes in a town.

The following graph represents a network of eight roads between five junctions, A, B, C, D and E, in a town.



(a) (i) Write down the adjacency matrix for this graph. [2]

(ii) Hence find the number of ways to travel from Junction A to Junction D using exactly four roads. The roads can be repeated. [3]

(This question continues on the following page)

(Question 1 continued)

On each day when there is snow, Leon drives a vehicle called a gritter to spread grit on the roads. The five junctions A, B, C, D and E must be connected by gritted roads, but the connections do not need to be direct. Leon's gritter dispenses $30\,\mathrm{kg}$ of grit per minute.

The following table shows the number of minutes it takes Leon to grit each road.

	A	B	C	D	E
A	-	5	-	4	9
B	5	-	7	2	8
C	-	7	-	5	-
D	4	2	5	-	6
E	9	8	-	6	-

(b) (i) State an appropriate algorithm to find a minimum spanning tree for the road network. [1]

(ii) Use this algorithm to determine which roads should be gritted. [3]

(iii) Find the minimum amount of grit Leon will need to grit these roads. [2]

On each day when there is no snow, Leon drives a street sweeper and must sweep **every** road in the network. The sweeper travels at the same speed as the gritter.

(c) (i) Find the minimum amount of time it takes Leon to sweep every road, beginning and ending his route at A. Justify your answer. [4]

(ii) State a valid route and justify why it minimizes the time taken. [1]

Leon learns that there will be maintenance work on the roads he must sweep. Each day, exactly one of the eight roads will be affected. The maintenance work doubles the time it takes to drive along that road. He must still begin and end his route at A.

(d) (i) Identify the road on which maintenance work will cause Leon to change his route. [1]

(ii) Determine the maximum amount of time the maintenance work will add to Leon's minimum route on this day. Justify your answer. [2]

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(Question 1 continued)

A new school opens in the town, and Leon's route now passes the school crossing twice each day. Each time Leon passes the crossing, there is a 25% chance he must stop for children.

If he must stop, the amount of time he waits can be modelled by a normal distribution with a mean of 3 minutes and a standard deviation of 0.8 minutes.

(e) (i) Show that the probability that Leon will not have to stop is 0.5625. [1]

(ii) Find the value of p and the value of q to complete the following probability distribution table for the number of times Leon will have to stop. [2]

Number of stops	0	1	2
Probability	0.5625	p	q

(iii) Find the probability that, on a given day, the total amount of time Leon will wait at the crossing is at least 4 minutes. [7]

2. [Maximum mark: 26]

This question relates to the growth rate of yeast colonies in different sugar concentrations.

Chaya is studying the growth response of a strain of yeast on nutrient plates with different initial sugar concentrations, C_0. After five days she measures the colony diameter, D, on each plate.

Her results are shown in the following table.

Plate	Initial sugar concentration C_0 (mol)	Colony diameter D after five days (mm)
P	1.0	3.1
Q	1.0	2.8
R	2.0	3.5
S	2.0	3.7
T	3.0	4.0
U	3.0	4.4
V	4.0	4.6
W	4.0	4.3

Chaya uses linear regression to find a regression line of the form

$$D = \alpha C_0 + \beta$$

with correlation coefficient $r = 0.950$.

(a) Find the value of α and the value of β. [2]

Chaya uses her regression line to estimate the colony diameter, after five days, of a plate with an initial sugar concentration of $10.0\,\text{mol}$.

(b) Comment on the appropriateness of this estimate. [2]

(This question continues on the following page)

(Question 2 continued)

Chaya's professor suggests that she investigate colony growth rate over time and recommends using a model

$$\frac{\mathrm{d}C}{\mathrm{d}t} = -aCD$$

$$\frac{\mathrm{d}D}{\mathrm{d}t} = bCD,$$

where time t is measured in hours, colony diameter D is measured in millimetres, and C, the sugar concentration at time t, is measured in moles.

The parameters a and b must be calculated from experimental data.

Chaya performs the experiment with an initial colony diameter of $0.1\,\mathrm{mm}$ and an initial sugar concentration of $3.0\,\mathrm{mol}$.

(c) (i) Using your equation from part (a) for initial sugar concentration $3.0\,\mathrm{mol}$, estimate the colony size after five days of growth. [1]

(ii) Show that in the model recommended by Chaya's professor, $bC + aD$ is constant. [3]

(iii) Given that after five days the sugar concentration is $0.1\,\mathrm{mol}$, find a in terms of b. [2]

(iv) Hence or otherwise, express C as a linear function of D. [1]

(v) Hence write $\frac{\mathrm{d}D}{\mathrm{d}t}$ as a function of b and D only. [1]

After studying the growth model against further experimental data, Chaya adjusts the parameters in the model to give

$$\frac{\mathrm{d}D}{\mathrm{d}t} = bD\left(3.08 - 0.8D\right),$$

and determines that an approximate value of $b = 0.025$.

(d) Assume an initial starting colony diameter of $0.1\,\mathrm{mm}$ and a step length, h, of 5 hours. Use Euler's method with Chaya's adjusted model to find the approximate colony diameter after 80 hours. [3]

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(Question 2 continued)

Chaya decides to use colony diameter at 80 hours with initial sugar concentration $3.0\,\text{mol}$ as a standard by which to assess different strains of the yeast species. She studies two samples, **A** and **B**, under the same conditions. She grows 15 colonies of each sample and measures the colony diameter of each at 80 hours.

	Sample A	**Sample B**
$\bar{x}\,\textbf{(mm)}$	3.45	3.66
$s_n^2\,\textbf{(mm}^2\textbf{)}$	0.0732	0.0411

(e) For each sample, find the unbiased estimate of the population variance. [2]

Chaya wants to test the hypothesis that the two samples come from populations with the same mean colony diameter at 80 hours. She assumes that the two samples come from populations with the same variance.

(f) (i) State an appropriate test. [2]

(ii) State the null and the alternative hypothesis for this test. [2]

(iii) Perform the test at the 5% significance level. [4]

(iv) State an additional assumption Chaya must make in using this test. [1]